An Introduction to Group Theory :-

A group G is set of elements $\{a, b, c, \dots\}$ including multiplication or composition rule.

i.e. if $a \in G$ and $b \in G$ then $ab \in G$.

It may also have a closure properties i.e. $ab = ba$. those group is called abelian and if $ab \neq ba$ then it is non-abelian.

Group always satisfy four properties.

1) Associativity :- multiplication rule is associative i.e. $(ab)c = a(bc)$

2) Identity element :- if $ae = ea = a$ then e is Identity element & it is unique for a group.

3) Inverse element :- there alway lies inverse of a $(a \in G)$ Such that $a^{-1}a = aa^{-1} = e$.

4) Order :- it is no. of elements in a group G.

Representation of a group :-

representation is a mapping that takes group elements $g \in G$ into linear operators F that preserve the composition rule of the group in the sense that,

$$F(a)\, F(b) = F(ab) \qquad \text{—①}$$

also $F(e) = I$ → preserves identity.

if $F \in H$ and $a, b \in G$ and if it satisfy eq ①

then it is said that, G is homomorphic to H.

Group parameter :-

like $F(x)$, group G is also a fn of one or more inputs called parameters. Say G is a group with element $g \in G$ are specified by finite set of parameter say n as.

$$\{\theta_1, \theta_2, \dots \theta_n\}$$

Then the group element g is

$$g = G(\theta_1, \theta_2, \dots, \theta_n)$$

here identity is defined as, $e = G(0, 0, \dots, 0)$

Lie Groups

Concider the group of infinite number of elements, it has a finite set of continiously varying parameters.

Concider group element $g = G(\theta_1, \theta_2, \dots, \theta_n)$ here the angle θ varyes continiously over a range $0 - 2\pi$. So G can be parameterized by a finite no of parameters, (here angle of rotation).

So, if a group G, (i) depends on a finite set of continious parameter, θ_i

(ii) Derivatives of group element w.r.t. all the parameter exist

then the group is known as, lie group.

Concider a group G with single element θ.

then, $g(\theta)\big|_{\theta=0} = e$. = identity.

Now generators of a group can be obtain by differentiating $g(\theta)$ w.r.t to its element θ at $\theta = 0$.

$$\therefore X = \frac{\partial g}{\partial \theta}\bigg|_{\theta=0}.$$

if there is n no. of paramete $(n \to \infty)$

then generator will be $X_i = \dfrac{\partial g}{\partial \theta_i}\bigg|_{\theta_i=0}$.

In rotation, length of vetor remains constant and after having $-\theta_i$ rot, it comes to same state as was before rotation.

So quantum mechanically rotation should not change the magnitude of a vector. hence it should be unitary and Hermition.

$$\therefore \; X_i = -i \frac{\partial g}{\partial \theta_i}\Big|_{\theta_i = 0}$$

this generator for finite θ_i allow to define a ~~gen~~ representation of a group (Say D).

$\therefore$ We can expand D in terms of $\epsilon\theta$ where ϵ very small +ve real no.

$$\therefore \; D(\epsilon\theta) = 1 + i\epsilon\theta X \qquad X = \text{generators.}$$

[not $\epsilon\theta$ = amount of rotation, X = generator of rotation]

here $\epsilon = \frac{1}{n}$, n = no. of rotation produced.

So for n no. of Succesive rotation,

$$D(\theta) = \lim_{n \to \infty} \left(1 + i\frac{\theta X}{n}\right)^n$$

$$= e^{i\theta X}$$

$$\therefore \; D^{\dagger}(\theta) = e^{-i\theta X^{\dagger}}$$

$$= e^{-i\theta X} \qquad \because X \text{ is hermitian}$$

[f_{ijk} is structural Constant.]

$$\therefore \; D^{\dagger}(\theta)\, D(\theta) = D(\theta)\, D^{\dagger}(\theta) = 1$$

Note:- generators are imp. because they forms the Vector Space. It means adding 2 generators third gen can be obtained belonging to same group. and even by multiplying by some scales, new obtained gen will be of Same group. Gen. follows Commutation relatn. $[X_i, X_j] = i f_{ijk} X_k$

Rotation Groups.

it preserves the lengths of a vectors. thus it requires the operator to be unitary.

we know if $a \in G$ & $b \in G$ then $ab \in G$.

So let R_1 and R_2 be two rotations. thus the Combination of these two rotation should be the element of Same group meaning combination should rise to new rotation.

$$\therefore R_3 = R_1 R_2 \quad \text{or} \quad R_3 = R_2 R_1$$

In general $\quad R_1 R_2 \neq R_2 R_1$

i.e. rotation is an non abelian operator.

However it follows the associative law,

$$R_1 (R_2 R_3) = (R_1 R_2) R_3 .$$

In Rotation group there lies a identity operation meaning operation giving no rotation. Similarly by opposite rotation one can give no rotation so there lies a inverse of a rotation.

Rotation Representation:-

Consider x_i be the two Dimensional p.v. which is rotated by an angle θ by Rotational operator (matrix) R_{ij} to transform it to net P.V. x'_j.

$$\therefore x'_j = R_{ij} x_i$$

In two dimension for θ angle rotation $R(\theta)$ matrix is given by

$$R(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

Such that

$$x_1' = \cos\theta\, x_1 + \sin\theta\, x_2$$

$$x_2' = -\sin\theta\, x_1 + \cos\theta\, x_2$$

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} \cos\theta\, x_1 + \sin\theta\, x_2 \\ -\sin\theta\, x_1 + \cos\theta\, x_2 \end{pmatrix}$$

$$= \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Consider the transpose of $R(\theta)$

$$[R(\theta)]^T = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Now $$R(\theta)\, R(\theta)^T = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$= \begin{pmatrix} \cos^2\theta + \sin^2\theta & -\cos\theta\sin\theta + \cos\theta\sin\theta \\ -\sin\theta\cos\theta + \sin\theta\cos\theta & + \sin^2\theta + \cos^2\theta \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

Thus $[R(\theta)]^T$ is a inverse of a $R(\theta)$. this can easily be proved if inverse of angle ie. $(-\theta)$ rotation is Produced ie $R(-\theta) = R(\theta)^T$

In group theory various groups are clarified according to determination of matrics

So $\det R(\theta) = \cos^2\theta + \sin^2\theta = 1$

However in general the det. is not equal to +1. for this case it is said that rot. matrix is a proper rotation.

Let us represent $R(\theta_1) \equiv R_1$

$\because$ $R_1 R_2$ represents the rotation $R_3 = R(\theta_1 + \theta_2)$

$\therefore$ $\det[R_3] = \det[R_1 R_2] = d[1][1] = I$

Thus one can say the matrix representation preserves the properties of a rotation group.

—— X —— X —— X ——

SO(N)

~~Som~~ The group SO(N) are special orthogonal $N \times N$ matrices. here special refers to the fact the matrices has determinant +1. SO(N) is a Subgroup of O(N) having $N \times N$ orthogonal matrix with any arbitary det.

Clearly Rotation belongs to SO(3) group. (in 3 Dim).

[Note for O to be Orthogonal matrix $OO^T = O^T O = I$ & for it to belong to SO(N) group $\det O = +1$]

There In 3 dimension there lies three types of rotation about x, y, z axis say ξ, ϕ & θ.

So Three rotation matrix is given by.

$$R_x(\xi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\xi & \sin\xi \\ 0 & -\sin\xi & \cos\xi \end{pmatrix} \quad R_y(\phi) = \begin{pmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{pmatrix}$$

$$R_z(\theta) = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

So here generator is defined as,

$$\frac{dR_x}{d\zeta} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\sin\zeta & \cos\zeta \\ 0 & -\cos\zeta & -\sin\zeta \end{pmatrix}$$

now setting $\zeta \to 0$

$$\left.\frac{dR_x}{d\zeta}\right|_{\zeta=0} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

Here $\quad J_x = -i \left.\frac{dR_x}{d\zeta}\right|_{\zeta=0} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$

Similarly $\quad J_y = -i \left.\frac{dR_y}{d\phi}\right|_{\phi\to 0} = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$

$$J_z = -i \left.\frac{dR_z}{d\theta}\right|_{\theta\to 0} = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Recall, J_x, J_y, J_z is a familiar angular momentum matrixes. This proves that angular momentum operator is a generator of rotation. So one can produce a small rotation using this generators say about z axis by $\epsilon\theta$ angle then

$$R_z(\epsilon\theta) = 1 + i J_z(\epsilon\theta).$$

from Quantum mechanics we also know,

$$[J_i, J_j] = i \epsilon_{ijk} J_k$$

where ϵ_{ijk} = Structural constant

= +1 for cyclic order of 123

= −1 " anti " " " 123

= 0 for random.

Say total of θ rotation is brought by a consecutive n no. of finite rotation $(n \to \infty)$.

then $$R_z(\theta) = \left[R_z\left(\frac{\theta}{n}\right)\right]^n$$

$$= \left[1 + i J_z \frac{\theta}{n}\right]^n$$

$$= 1 + i J_z \theta + \ldots\ldots$$

$$= e^{i J_z \theta}$$

$$\therefore \quad R(\vec{\theta}) = e^{i \vec{J} \cdot \vec{\theta}}$$

Unitary Group.

Unitary operators preserve the inner products, meaning, that a unitary transformation leaves the probability for different ~~invariant~~ transition among the states ~~invari~~ unaffected. That is quantum physics is invariant under unitary transformation. Also unitary operators commutes with Hamiltonian H.

$$[U, H] = 0.$$

$U(N)$ consists of $N \times N$ unitary matrices. Special unitary groups denoted by $SU(N)$ are $N \times N$ unitary matrices with +ve unit determinant. dimension of $SU(N)$ is given by $N^2 - 1$

and is same to no. of generators.

i.e. $SU(2)$ has $2^2-1=3$ generators.

$SU(3)$ " $3^2-1=8$ "

and the rank of $SU(N)$ is $N-1$

i.e. rank $SU(2)$ is $2-1=1$

" $SU(3)$ " $3-1=2$.

rank gives the no. of operators simultaneously diagonalized

Concider a simplest unitary operator $U(1)$ represented by a ~~Sypa~~ Symmetry in angle θ.

Say $U = e^{-i\theta}$.

here $U_1U_2 = e^{-i\theta_1}e^{-i\theta_2} = U_2U_1$

$\boxed{U_1U_2 = U_2U_1}$ — ~~ablibian~~ abelian.

Note :- Many lagrangian in QFT is invariant under $U(1)$ transformation.

Concider $z = re^{i\alpha}$ be any arbitary number to will if multiplied by $e^{i\theta}$ then,

$$e^{i\theta}z = re^{i(\theta+\alpha)}$$

So new complex number have same length with increased angle.

For complex lagrangian $\mathcal{L} = \partial_\mu\phi^+\partial^\mu\phi - m^2\phi^+\phi$.

It is invariant under transformation

$$\phi \to U\phi = e^{-i\theta}\phi.$$

from noethers theorym there lies a symmetry and in this case it us $U(1)$ Symmetry.

It will be shown in electrodynamics, gauge boson (Force mediating particles) associated with ϕ U(1) symmetry is photon.

Unitary symmetry U(1) results in conservation of various quantum numbers. say U(1) symmetry is associated with quantum no. a then $U = e^{-ia\theta}$

Another significance is that Hamiltonian is invariant under transformation $e^{-ia\theta} H e^{ia\theta}$, i.e.

$$UHU^{+} = H.$$

In this case adjoint of U i.e U^{+} is inverse of U

i.e. $UU^{+} = U^{+}U = I$

$\therefore \; U(-\theta)\,U(\theta) = U(\theta)\,U(-\theta) = I$

Such symmetry exist with conversion of lepton & baryon numbers.

Say, an element of group U(1) is a complex number of unit length as,

$$U = e^{-i\theta}$$

——— X ——— X ——— X ———

Now consider a unitary group U(2). which is a set of 2X2 unitary matrics.

so $U^{+}U = UU^{+} = I$

Now we are interested with sub group of U(2) with determinant +1. This group is SU(2). In this case pauli matrix is the generators.

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Now there comes the rank of SU(2) which is 1 which rep. the no. of diagonalized operator. which in this case is σ_3.

Here we have chosen our generator as $\frac{1}{2}\sigma_i$ and the algebra is the familiar comutation rel^n. satisfied by paulis matrix is

$$\boxed{\left[\frac{\sigma_i}{2}, \frac{\sigma_j}{2}\right] = i\,\epsilon_{ijk}\,\frac{\sigma_k}{2}}$$

Also pauli matrix does not commute i.e. $U_1U_2 \neq U_2U_1$

So SU(2) is non abelian.

An element of SU(2) Group can be written as,

$$U = e^{i\sigma_j \alpha_j/2}$$

where σ_i is one of pauli matrix and α_j is a number. to understand the use of 1/2, let us concider SU(2) & SO(3) correspondance. ~~both~~ we know SO(3) ~~&~~ ~~SU(2)~~ reserves the length of a vector.

if $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$ Consider a matrix $\vec{\sigma}.\vec{r}$

$$\therefore \vec{\sigma}.\vec{r} = \sigma_x x + \sigma_y y + \sigma_z z$$

$$= \begin{pmatrix} 0 & x \\ x & 0 \end{pmatrix} + \begin{pmatrix} 0 & -iy \\ iy & 0 \end{pmatrix} + \begin{pmatrix} z & 0 \\ 0 & -z \end{pmatrix}$$

$$\boxed{\vec{\sigma}.\vec{r} = \begin{pmatrix} z & x-iy \\ x+iy & -z \end{pmatrix}}$$

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$\therefore$ det $\vec{\sigma}.\vec{r} = \begin{pmatrix} z & x-iy \\ x+iy & -z \end{pmatrix} = -z^2 - (x^2+y^2)$

$$= -|\vec{x}|^2 .$$

also $\vec{\sigma}.\vec{r}$ is hermitian with zero trace. Now consider unitary transformation on this matrix.

$$\therefore U(\vec{\sigma}.\vec{r})U^{+} = \sigma_x (\vec{\sigma}.\vec{r}) \sigma_x$$

$$= \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} z & x-iy \\ x+iy & -z \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} -z & x+iy \\ x-iy & z \end{pmatrix}$$

this still has zero trace & is Hermitian. with Same determinant. this means SU(2) also preserves length.

Now consider the SU(2) transformation on two Component spiner, like

$$\psi = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

where $x = \frac{1}{2}(\beta^2 - \alpha^2)$; $y = \frac{i}{2}(\alpha^2 + \beta^2)$; $z = \alpha\beta$.

So SU(2) transformation on $\psi = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$ is just like that of SO(3) transformation of $\bar{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$. So SU(2) transformation is associated with three angles $\alpha \cdot \beta \,\&\, \gamma$ where 1/2 the rotation produced by SU(2) is equivalent to SO(3). So for arbitary angle α. a transformation generating this rot. along x axis in SU(2) is given by,

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$$U = \begin{pmatrix} \cos\alpha/2 & i\sin\alpha/2 \\ i\sin\alpha/2 & \cos\alpha/2 \end{pmatrix}$$

Similarly in y axis it is.

$$U = \begin{pmatrix} \cos\beta/2 & \sin\beta/2 \\ -\sin\beta/2 & \cos\beta/2 \end{pmatrix}$$

& for rot in z axis $U = \begin{pmatrix} e^{i\gamma/2} & 0 \\ 0 & e^{-i\gamma/2} \end{pmatrix}$

∴ one can rep. all above by two parameter

$$U = \begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix}$$

So $SU(2)$ det $U = +1$ which req

$$a^2 + b^2 = 1$$

This symmetry is imp. in case of weak inte. in case of electro weak interaction, gauge-bosons corresponds to W & Z bosons that carry weak int.

Now Concider $SU(3)$, here we will have 8 generators and its rank is 2. It is of importance in case of study of quarks and QCD. Thes generators is called Gell-Mann matrices.

$$\lambda_1 = \begin{pmatrix} \sigma_1 & 0 \\ 0 & 0 \end{pmatrix} \cdot \lambda_2 = \begin{pmatrix} \sigma_2 & 0 \\ 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}; \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}; \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Here λ_3 and λ_8 is a diagonal matrix. these matrix is trace less and it satisfy the commutation relation.

$$[\lambda_i, \lambda_j] = 2i \sum_{k=1}^{8} f_{ijk} \lambda_k$$

where structure coefficent is such that

$$f_{123} = 1 \quad f_{147} = f_{165} = f_{246} = f_{257} = f_{345} = f_{376} = 1/2$$

$$f_{458} = f_{687} = \frac{\sqrt{3}}{2} \quad \text{~~& rest is zero~~}.$$

Casimir Operators.

It is a non linear fn of generators of a group that commuts with all of other generators. the no. of casimrer operator is given by its rank.

Say for SU(3)

$J^2 = J_x^2 + J_y^2 + J_z^2$ is a Casimier operator

& J^2 Commutes with all J_x J_y & J_z.

This operator is in variant. i.e. it is a multiple of group identity element.

Discrete Symmetries and Quantum Numbers :-

Additive and Multiplicative Quantum Numbers :-

$$\sum n_i = \sum m_i$$

n_i = quantum no before reaction

m_i = " " after "

then it is additive Quantum number.

if $\prod n_i = \prod m_i$ then it is multiplicative.

for Composite system, $\sum n_i$ or $\prod n_i$ is if conserved the It represent a Symmetry of a system.

Parity

if the potential V is symmetric about origin such that $V(-x) = V(x)$ & if $\psi(x)$ be soln of schrödinger eqn the $\psi(-x)$ is also the soln. ie.

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi(-x) + V(-x)\,\psi(-x) = E\,\psi(-x).$$

Also both $\psi(-x)$ & $\psi(x)$ solves to same eigenvalue E. this requires that $\psi(x)$ & $\psi(-x)$ is different by a constant parameter, say α.

$$\psi(x) = \alpha\,\psi(-x) \quad -(1)$$

interchanging $x \to -x$

$$\psi(-x) = \alpha\,\psi(x) \quad -(11)$$

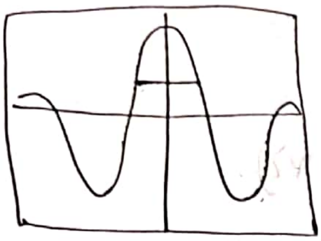
Therefore $\alpha^2 = 1$ from (1) & (11)

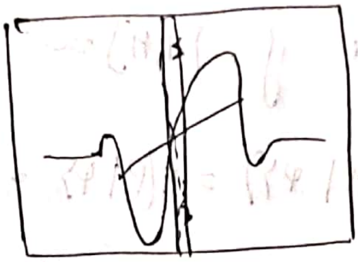
or $\alpha = \pm 1$

ie. $\psi(-x) = -\psi(x)$ → we say W.F has odd parity.

or $\psi(-x) = \psi(x)$. " " " " even "

even power leads to even parity & odd power leads to odd parity in W.F.





this leads us to one parity operator P with eigen value $+1$ or -1 and gives the test of odd or even parity.

applying the parity operator twice gives the original wave fn

$$P^2 \psi(x) = P\psi(-x) = \psi(x)$$

$$\therefore \quad P^2 = 1$$

Also. if $|\psi\rangle$ be the wave fn. then

$$P|\psi\rangle = \pm|\psi\rangle$$

if $|\psi\rangle$ is an angular momentum state with the momentum L then $P|L, m_z\rangle = (-1)^L |L, m_z\rangle$

ie. parity $= (-1)^L$. also if Parity Op. Commute with H then parity is Conserved.

$$[P, H] = PH - HP = 0.$$

Consequence of this is that for a state with parity α will reain same with evolution of time.

Also if $|\psi\rangle$ is nondegenerate eigenstate of H with eigen value E then,

$$P(H|\psi\rangle) = P(E|\psi\rangle) = E\,P(|\psi\rangle).$$

However if $[P,H] = 0$ then

$$P(H|\psi\rangle) = H(P|\psi\rangle = E(P|\psi\rangle).$$

So eigen state of H is also the eigenstate of H. So eigen state of P is $\alpha = \pm 1$ and is a important tool to identify the particles fermions and bosons. Thus it is a property of a particle and is intrensic parity of the particle.

Notes $\rightarrow$ Particle with spin - 1/2 have positive parity. ie electron and quarks has $\alpha = +1$.

Antiparticles with spin 1/2 have -ve parity therefore positron has $\alpha = -1$.

Bosons have same parity for both particles and antiparticles.

$\rightarrow$ let $|\psi\rangle = |b\rangle|a\rangle$ be the composite system with parity of $|a\rangle$ & $|b\rangle$ be P_a & P_b respectively then the parity of the composite system is $\boxed{P_\psi = P_a P_b}$ ie. parity is multiplicative in nature.

One can construct a new parity operator by combining P with one of the Conserved charges of the Standard model. Such as (1) elec. char. op. Q (2) Lepton no. L (3) Baryon no. B.

usually parity is conserved in the electromagnetic & strong interactions but is not conserved in weak interaction.

Particles are labeled as, $\boxed{J^P = \text{spin}^{\text{parity}}}$

For spin-0 particle with negative parity is called pseudoscaler. eg. π, k mesons etc. it is rep. by 0^-

For spin-0 particle with +ve parity is called scaler. eg. Higgs Boson which is a field believed to provide mass generation. It is rep. by 0^+.

A vector boson has spin-1 & -ve parity (1^-). eg. photon similarly pseudo vector has spin -1 and +ve parity.

parity is not conserved in weak interaction & is called Parity violation. it is observed in weak decay of Co^{60}. it is also found in weak decay of θ & τ mesons.

$\theta^+ \to \pi^+ \pi^0$

$\tau^+ \to \pi^+ \pi^- \pi^+$

} different final state of the $\overset{+}{\theta}$ & $\overset{+}{\tau}$ suggest that they were dif. particle. but parity violation resolves that they are same particle known as $K^{\pm}$ mesons.

Charge Conjugation.