Time Series

***
Q. What is time series? Explain the various components of time series. **

→ A series of observations recorded in accordance with the time of occurance is known as time series data. Such data are very important in business, commerce, trade etc for forecasting the future demand and also any change in future.

There are four components of time series data. The components are:

(i) Secular Trend or Simply Trend. (T)
(ii) Seasonal Variation (S)
(iii) Cyclical Fluctuation. (C)
(iv) Irregular or Random Movement. (I)

02
(i) Secular Trend → Secular Trend is a smooth, regular and long term movement exhibiting the tendency of either growth or decline over a period of time. The population growth together with advances in technology and methods of business organisation are the main factor for the growth or upward trend in most of the economic and business data. The decline of downward trend may be due to the decrease in demand of the product, or a substitute taking its place, or difficulty in obtaining raw-materials etc.

(ii) Seasonal Variation → Seasonal Variation represents a type of periodic movement, wh the period is not longer than one year. Th up and down, movements of business activities, ~~recurring~~ (আবর্তক) with remarkable regularity year after year is attributable (আরোপনীয়) to the presence of seasonal variations. The factors which cause this type of variations are the climatic (আবহাওয়া) changes of the different seasons, such as changes in rainfal temperature, humidity etc. and the customs and habits which people follow at different part of the year.

(iii) Cyclical Fluctuation → Cyclical Fluctuatio is another ~~a~~ type of periodic movement where the period is ~~not~~ longer than a year. Cyclical fluctuation is found to exists in most of the economic and business time serie where it is known as Business Cycle. Prosper, decline, depression and recovery are the four ~~phag~~ phases.

(iv) Irregular or Random Movement → Irregular or Random Movements are such variations which are caused by factors of an erratic nature. These are completely unpredictable or caused by some unforeseen events as war, flood, earthquake, strike, lock-out, etc. Random moveme do not revail any pattern of the repeatative tendency and may be ~~consereede~~ consedered as Residual Variation.

Date- 04/10/17. Day-6

P-641
[redacted]

16.11 (a) Calculate the five-yearly Moving Average of the following:

Year	1950	1951	1952	1953	1954	1955	1956	1957	1958
Values	105	115	100	90	80	95	85	75	60
	1959	1960	1961	1962	1963	1964	1965	1966	
	65	70	58	55	53	60	52	50	

(b) Illustrate with four examples, how the four-yearly Moving Average can be calculated.

Sol^n

Calculations for 5-yearly Moving Average

Year	Value (Y)	5 year moving Total (Centered)	5 year moving average
1950	105	-	-
1951	115	-	-
1952	100	490	98
1953	90	480	96
1954	80	450	90
1955	95	425	85
1956	85	395	79
1957	75	380	76
1958	60	355	71
1959	65	328	65.6
1960	70	308	61.6
1961	58	301	60.2
1962	55	296	59.2
1963	53	278	55.6
1964	60	270	54
1965	52	-	-
1966	50	-	-

Calculations for 4-yearly Moving Average.

Year	Value	4 year moving total (Not Centered)	2-items moving total (Centered)	4 year moving average
		–	–	
1950	105	–	–	
1951	115	410		
1952	100	385	795	99.375
1953	90	365	750	93.750
1954	80	350	715	89.375
1955	95	335	685	85.625
1956	85	315	650	81.250
1957	75			
1958	60	–	–	
		–	–	

Exercise-11) For the following series of observation verify that the 4-year
P-678) centred moving averages is equivalent to a 5 year weighted moving average with weights 1, 2, 2, 2, 1 respectively:

Year	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974
Sales Rs '0000	2	6	1	5	3	7	2	6	4	8	3

Solⁿ:-

Year	Sales (Rs '0000)	4 year moving total (not Centered)	2 items moving total (Centered)	4 year moving avg.
1964	2	–	–	–
1965	6	–	–	–
1966	1	14	29	3.625
1967	5	15	31	3.875
1968	3	16	33	4.125
1969	7	17	35	4.375
1970	2	18	37	4.625
1971	6	19	39	4.875
1972	4	20	41	5.125
1973	8	21	–	–
1974	3	–	–	–

Year	Sales (Rs'0000)	5 year weighted moving total (Centered)	5 year weighted moving average
1964	2	–	–
1965	6	–	–
1966	1	→ 29	3.625
1967	5	→ 31	3.875
1968	3	→ 33	4.125
1969	7	→ 35	4.375
1970	2	→ 37	4.625
1971	6	→ 39	4.875
1972	4	→ 41	5.125
1973	8	–	–
1974	3	–	–

$(2\times1+6\times2+1\times2+5\times2+3\times1)=29$

Thus 4 year centered moving average is equivalent to 5 year weighted moving average when the weights are 1, 2, 2, 2, 1 (Proved).

Example-16.18) ~~Exer~~ Fit a straight line trend by the least squares $\boxed{\text{P-648}}$ method to the following figures of production of a sugar factory:

Year	1969	1970	1971	1972	1973	1974	1975
Production ('000 tons)	76	87	95	81	91	96	90

Estimate the production for 1976.

→ Let $y=a+bx$ be the st. line trend equation with origin at the year 1972 and unit of x = 1 year. In order to find out the values of 'a' and 'b' by using least ~~squres~~ squares method, the normal equations are

$$\Sigma y = an + b\Sigma x \longrightarrow (1)$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 \longrightarrow (2)$$

Now we prepare the following table :-

Year	Production ('000 tones) (Y)	x	x^2	xy
1969	76	-3	9	-228
1970	87	-2	4	-174
1971	95	-1	1	-94
1972	81	0	0	0
1973	91	1	1	91
1974	96	2	4	192
1975	90	3	9	270
	616	0	28	56.

Putting these values into the normal eqⁿs, we get

$$616 = a \times 7 + b \times 0 \quad \dots \quad (3)$$

$$56 = a \times 0 + b \times 28 \quad \dots \quad (4)$$

From (3), $a = \frac{616}{7} = 88$

From (4), $b = \frac{56}{28} = 2.$

Thus the st. line trend equation is $y = 88 + 2x$ ('000 tones) with origin 1972 and unit of $x = 1$ year

In 1976, $x = 4$

Then $y = 88 + 2 \times 4$

$= 88 + 8$

$= 96$ ('000 tones) (Ans)

Date- 06/10/17 Day-7

P-651 Ex.16.20

Using 1964 as the origin, obtain a st. line trend equation by the method of least squares. Find the trend value of the missing year 1961.

Year	1960	1962	1963	1964	1965	1966	1969
Value	140	144	160	152	168	176	180

~~Solution~~

Solution

Let $y = a+bx$ be the st. line trend equation with origin at the year 1964 and unit of $x = 1$ year. In order to find out the values of the constants 'a' and 'b' by by least squares method the normal equations are:

$$\Sigma y = an + b\Sigma x \quad \ldots \quad (1)$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 \quad \ldots \quad (2)$$

Now we prepare the following table:

Year	Value(Y)	x	x^2	xy
1960	140	-4	16	-560
1962	144	-2	4	-288
1963	160	-1	1	-160
1964	152	0	0	0
1965	168	1	1	168
1966	176	2	4	352
1969	180	5	25	900
	1120	1	51	412

Putting these values into the normal eq^{n}s, we get,

$$1120 = a \times 7 + b \times 1 \quad \ldots \quad (3)$$

$$412 = a \times 1 + b \times 51 \quad \ldots \quad (4)$$

or, $7a + b = 1120 \quad \ldots \quad (3) \times 1$

$a + 51b = 412 \quad \ldots \quad (4) \times 7$

$7a + b = 1120$

$7a + 357b = 2884$

$- \quad - \quad -$

$-356b = -1764$

$\therefore b = \dfrac{1764}{356}$

$\therefore b = 4.96$

From (3), $7a + 4.96 = 1120$

$\therefore 7a = 1120 - 4.96$

$\therefore a = \dfrac{1115.04}{7} = 159.29$

Thus the st. line trend eq^n is $y = 159.29 + 4.96x$ with origin at the year 1964 and unit of $x = 1$ year.

In 1961, $x = -3$

Then $y = 159.29 + 4.96 \times (-3)$

$= 159.29 - 14.88$

$= 144.41$

Thus trend value in the missing year 1961 is 144.41. (Ans).

Ex. 16.19 / P-649 Fit a st. line trend equation by the method of least squares from the following data and then estimate the trend value for the year 1985:

Year	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980
Value ('000)	65	80	84	75	77	71	76	74	70	68

Soln:- Let $y = a + bx$ be the st. line trend eqn with origin at the mid point of 1975 and 1976 and unit of x = 6 months. In order to find out the values of the constant 'a' & 'b' by using least squres method the normal eqn's are:

$$\Sigma y = an + b\Sigma x \quad \ldots \quad (1)$$
$$\Sigma xy = a\Sigma x + b\Sigma x^2 \quad \ldots \quad (2)$$

We now prepare the following table:

Year	value('000)(y)	x	x^2	xy
1971	65	-9	81	-585
1972	80	-7	49	-560
1973	84	-5	25	-420
1974	75	-3	9	-225
1975	77	-1	1	-77
1976	71	1	1	71
1977	76	3	9	228
1978	74	5	25	370
1979	70	7	49	490
1980	68	9	81	612
	740	0	330	-96

Putting these values into the normal eqⁿs, we get,

$$740 = a \times 10 + b \times 0 \quad \ldots \quad (3)$$
$$-96 = a \times 0 + b \times 330 \quad \ldots \quad (4)$$

From (3), $a = \frac{740}{10} = 74$

From (4), $b = \frac{-96}{330} = -0.29$

Thus the st. line trend eqⁿ is $y = 74 - 0.29x$ with origin at the mid point of 1975 and 1976 and unit of $x = 6$ months. (Ans.)

In 1985, $x = 19$.

Then $y = 74 - 0.29 \times 19$

$= 74 - 5.51 = 68.49$ ('000)

Thus the trend value in 1985 is 68.49('000)

Example- 16.21 [P- 651] Fit a second degree polynomial to the following data:

Year	1882	1883	1884	1885	1886	1887	1888	1889	189
Price Index	84	82	76	72	69	68	70	72	73

Solⁿ:- Let, $y = a + bx + cx^2$ be the equation of second degree polynomial (Parabola) with origin at the year 1886 and unit of $x = 1$ year. In order to find out the ~~Constanstant~~ constants 'a', 'b' & 'c' by using least squares method, the

the normal eqⁿs are :

$$\Sigma y = an + b\Sigma x + c\Sigma x^2 \longrightarrow (1)$$
$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3 \longrightarrow (2)$$
$$\Sigma x^2y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4 \longrightarrow (3)$$

Now we prepare the following table :

Year	Price Index (y)	x	x^2	x^3	x^4	xy	x^2y
1882	84	-4	16	-64	256	-336	1344
1883	82	-3	9	-27	81	-246	738
1884	76	-2	4	-8	16	-152	304
1885	72	-1	1	-1	1	-72	72
1886	69	0	0	0	0	0	0
1887	68	1	1	1	1	68	68
1888	70	2	4	8	16	140	280
1889	72	3	9	27	81	216	648
1890	73	4	16	64	256	292	1168
	666	0	60	0	708	-90	4622

Putting these values into the above eqⁿs, we get

$$666 = a\times 9 + b\times 0 + c\times 60 \longrightarrow (4)$$
$$-90 = a\times 0 + b\times 60 + c\times 0 \longrightarrow (5)$$
$$4622 = a\times 60 + b\times 0 + c\times 708 \longrightarrow (6)$$

From (5), $b = \frac{-90}{60} = -1.5$

Again,

$$9a + 60c = 666 \text{ ——— } (4)\times 60$$
$$60a + 708c = 4622 \text{ ——— } (6)\times 9$$

$$540a + 3600c = 39960$$
$$540a + 6372c = 41598$$
$$\underline{- \quad\quad - \quad\quad\quad -}$$
$$-2772c = -1638$$

$$\therefore c = \frac{1638}{2772} = 0.59$$

From (4), $9a + 60 \times 0.59 = 666$

or, $9a = 666 - 35.4$

$\therefore a = \frac{630.6}{9} = 70.07$

Thus the equation of second degree polynomial is $y = 70.07 - 1.5x + 0.59x^2$ with origin at the year 1886 and unit of $x = 1$ year.

(Ans).

Example. 16.22

P-652

Fit an exponential trend $y = ab^x$ to the following data by the method of least squares and find the trend values for the years 1944-48:

Year(x)	1942	1943	1944	1945	1946	1947	1948
Sales(y)	87	97	113	129	202	195	193

Soln:- Here, [redacted] $y = ab^x$ be the exponential trend with origin at the year 1945 and unit of $x = 1$ year.

Taking logarithms on both sides, we get

$\log y = \log a + x \log b$

or, $Y = A + Bx$, where, $Y = \log y$,

$A = \log a$

and, $B = \log b$

Now, $Y = A + Bx$ is the eqn of a st. line. In order to find out the constants A and B by least squares method, the normal eq's are

$$\Sigma Y = An + B\Sigma x$$

$$\Sigma xY = A\Sigma x + B\Sigma x^2$$

Now we preparce the following table :-

Year	Sales(y)	Y = log y	x	x^2	xY
1942	87	1.9395	−3	9	−5.8185
1943	97	1.9868	−2	4	−3.9736
1944	113	2.0531	−1	1	−2.0531
1945	129	2.1106	0	0	0
1946	202	2.3054	1	1	2.3054
1947	195	2.2900	2	4	4.5800
1948	193	2.2856	3	9	6.8568
		14.9710	0	28	1.8970

Putting these values into the above normal eq^n we get ——

$$14.9710 = A\times 7 + B\times 0 \quad \text{——— (1)}$$

$$1.8970 = A\times 0 + B\times 28 \quad \text{——— (2)}$$

From(1), $A = \dfrac{14.9710}{7} = 2.1387$

From (2), $B = \dfrac{1.8970}{28} = 0.0678$

$\therefore$ $A = \log a = 2.1387$

$\therefore$ $a = \text{Antilog } 2.1387 = 137.6$

Again, $B = \log b = 0.0678$

$\therefore$ $b = \text{Anti Log } 0.0678 = 1.169$

Thus the eq^n of exponential trend is $y = (137.6)(1.169)^x$ with origin at the year and unit of $x = 1$ year (Ans).

In 1944, $x = -1$

Then the trend value is $y = (137.6)(1.169)^{-1}$

$= 117.70$ (Ans)

In 1945, $x = 0$

Then the trend value is $y = (137.6)(1.169)^{0}$

$= 137.6$ (Ans)

In 1946, $x = 1$,

Then the trend value is $y = (137.6)(1.169)^{1}$

$= 160.85$ (Ans)

In 1947, $x = 2$.

The trend value is $y = (137.6)(1.169)^{2}$

$= 188.03$ (Ans)

In 1948, $x = 3$,

The trend value is $y = (137.6)(1.169)^{3}$

$= 219.81$ (Ans)

— o —

Process of finding Monthly trend equation from ~~ea~~ yearly equation:

A. Odd No. of years.

1. $y = a + bx$
[origin: 1950 and unit of $x = 1$ year]

2. The monthly trend equation is
$$y = a + \frac{b}{12}x$$
[origin: 30th June 1950 and unit of $x = 1$ month]

3. The monthly trend equation for a specific month is
$$y = a + \frac{b}{12}\left(x + \frac{1}{2}\right)$$
[origin: July 1950 and unit of $x = 1$ month]

B. Even No. of years.

1. $y = a + bx$
[origin: At the mid point of 1950 and 1951 and unit $x = 6$ months]

2. The monthly trend equation is
$$y = a + \frac{b}{6}x$$
[origin: 31st Dec. 1950 and unit of $x = 1$ month]

3. The monthly trend equation for a ~~spef~~ specific month is
$$y = a + \frac{b}{6}\left(x + \frac{1}{2}\right)$$
[origin: Jan. 1951 and unit of $x = 1$ month]

C. In case of Annual Totals

$$Y = A + Bx$$
$$\rightarrow y = \frac{A}{12} + \frac{B}{12}x$$
$$= a + bx.$$

Ex-16:23
P-655 The trend equation fitted to annual aver[age] sales is given by $y = 230 + 20x$, unit of x = one year, origin — 1960. Adjust the trend equ[ation] for finding the monthly trend values, and fi[nd] trend values for the months of January–Ma[rch] 1972.

Solⁿ:- The trend equation fitted to annual average sales is given by $y = 230 + 20x$ with origin at the 1960 and unit of x = 1 year.

The monthly trend equation is

$$y = 230 + \frac{20}{12} x$$

or, $y = 230 + 1.667x$

(origin: 30th June 1960 and unit of x = 1 month)

Now the monthly trend equation for a specific month is

$$y = 230 + 1.667\left(x + \frac{1}{2}\right)$$

$$= 230 + 1.667x + 0.833$$

$$= 230.833 + 1.667x$$

(origin: July 1960 and unit of x = 1 month)

(Ans.)

In Jan. 1972, $x = 138$

The trend value is $y = 230.833 + 1.667 \times 138$

$= 460.87$ A.

In Feb. 1972, $x = 139$.

The trend value is $y = 230.833 + 1.667 \times 139$

$= 462.54$ (Ans)

In March, 1972, $x = 140$.

The trend value is $y = 230.833 + 1.667 \times 140$

$= 464.213$.

Ex. 16.25 Fit a st. line used trend to the following data P-656 and show how you would obtain the monthly trend values from the trend line fitted to the yearly values, and obtain two such monthly values:

Year	1946	1947	1948	1949	1950	1951	1952	1953	1954
Average Monthly Profits (Million Rs)	6.3	7.4	9.3	7.4	8.3	10.6	9.0	8.7	7.9

Soln:- Let $y = a + bx$ be the straight line trend eqn with origin at the year 1950 and unit of $x = 1$ year. In order to find out the ~~con~~ constants 'a' and 'b' by least squares method the normals eqns are:

$$\Sigma y = an + b\Sigma x$$
$$\Sigma xy = a\Sigma x + b\Sigma x^2$$

Now we prepare the following table:

Year	Average monthly Profit (millions Rs.)	x	x^2	xy
1946	6.3	-4	16	-25.2
1947	7.4	-3	9	-22.2
1948	9.3	-2	4	-18.6
1949	7.4	-1	1	-7.4
1950	8.3	0	0	0
1951	10.6	1	1	10.6
1952	9.0	2	4	18.0
1953	8.7	3	9	26.1
1954	7.9	4	16	31.6
	74.9	0	60	12.9

Putting these values into the normal eqns, we ge

$$74.9 = a \times 9 + b \times 0 \quad \text{———} (1)$$
$$12.9 = a \times 0 + b \times 60 \quad \text{———} (2)$$

From (1), $a = \frac{74.9}{9} = 8.33$

From (2), $b = \frac{12.9}{60} = 0.215$

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Thus the st. line trend eq^n is $y = 8.33 + 0.215x$ with origin at the year 1950 and unit of x = 1 year.

The monthly trend eq^n is

$$y = 8.33 + \frac{0.215}{12} x$$

$$= 8.33 + .018x,$$

[origin: 30th june 1950 and unit of $x = 1$ month]

Now the monthly trend eq^n for a specific month is $y = 8.33 + 0.018\left(x + \frac{1}{2}\right)$

$$= 8.33 + 0.018x + .009$$

$$= 8.339 + 0.018x$$

[origin: July 1950 and unit of x = 1 month] Ans.

Suppose we interested to find out the trend values of March 1951 and Nov 1949.

In March 1951, $x = 8$

The trend value is

$$y = 8.339 + 0.018 \times 8$$

$$= 8.483 \text{ (Million Rs.) Ans.}$$

In Nov. 1949, $x = -8$

The trend value is

$$y = 8.339 + 0.018 \times -8$$

$$= 8.195 \text{ (Million Rs.)}$$

Ans

Ex. 16.26 P-657	Fit a st. line trend by the method of ... squares to the following series of observ...

Year	1943	1944	1945	1946	1947	1948	1949	1950
Y	683	687	678	665	656	689	691	~~69~~ 6

Y being the production in thousand tons. From the fit... line, indicate how you would find monthly tre... values.

Soln:- [Note that the given values of Y are annual totals]

Let the equation of st. line trend for the annual totals be $y = a + bx$ with origin at the mid point of 1946-1947 and unit of x = 6 months ; $y = Y$-

The normal equations for finding constants 'a' & 'b' are

$$\Sigma y = an + b\Sigma x$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2.$$

Now, prepare the following table:-

Year	Y	$y = Y - 670$	x	x^2	xy
1943	683	13	-7	49	-91
1944	687	17	-5	25	-85
1945	678	8	-3	9	-24
1946	665	-5	-1	1	5
1947	656	-14	1	1	-14
1948	689	19	3	9	57
1949	691	21	5	25	105
1950	698	28	7	49	196
$n = 8$	5447	87	0	168	149

Substituting the values of in the normal equation

$87 = a(8) + b(0)$ or, $8a = 87$

$149 = a(0) + b(168)$ $\quad 168b = 149$

Therefore,

$a = 10.875$, $b = 0.8867$

Hence, the st. line trend is

$y = a + bx$ i.e, $Y - 670 = 10.875 + 0.887x$

or, $Y = 680.875 + 0.887x$

Thus the st. line trend equation fitted to the annual totals is $Y = 680.875 + 0.887x$ with origin at the ~~year~~ mid of the year 1946 & 1947 and unit of x = 6 months, unit of Y = thousand tons.

The ~~monthly trend eq^n is~~ ∴ The trend line fitted to monthly average is

$$Y = \frac{680.875}{12} + \frac{0.887}{12}x$$

or, $Y = 56.74 + 0.074x$

[origin - at the mid of the year 1946 & 1947, unit of x = 6 months, Y = monthly average production]

The monthly trend eq^n is

$$Y = 56.74 + \frac{0.074}{6}x$$

$$= 56.74 + 0.0123x$$

[origin - 31st Dec, 1946 & unit of x = 1 month & unit of Y = monthly production in thousand tons]

The monthly trend eq^n for a specific month is

$$Y = 56.74 + 0.0123\left(x + \frac{1}{2}\right)$$

$$= 56.74 + 0.0123x + 0.00615$$

$$= 56.75 + 0.0123x$$

[origin - Jan. 1947, unit of x = 1 month, Y = monthly production in tho tons)]

Suppose we interested to find out the trend value of Aug 1944 and May 1952,

11111 + 12 + 12 + 1 = 30 - 1 = -29

In Aug, 1944, $x = -29$,

$$Y = 56.75 + 0.0123(-29)$$
$$= 56.75 - 0.3567$$
$$= 56.3933 \text{ thousand tons.}$$

In May 1952, $x = 64$

$$Y = 56.75 + 0.0123(64)$$
$$= 56.75 + 0.7872$$
$$= 57.5372 \text{ thousand tons.}$$

111111111111 + 12 + 12 + 12 + 12 + 11111 = 65 - 1 = 64

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Sesonal Variations	

Day-10 Trend ratio Method Date- 12/10/17

P-
Example- [illegible]
Book: N. G. Das, Statistical Methods

Let $y = a + bx$ be the st. line trend equation with origin at the mid point of quarters II & III of 1978 unit of $x = \frac{1}{2}$ quarter. In order to find out the constants 'a' and 'b' by the method of least squres the normal equations are:

$$\Sigma y = an + b\Sigma x$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2$$

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Now we prepare the following table :-

Year/quarter	Time series(y)	x	x^2	xy	Trend value(T)	Tre rat
1977 I	165	-11	121	-1815	155.7	106
II	135	-9	81	-1215	152.9	88
III	140	-7	49	-980	150.2	93
IV	180	-5	25	-900	147.4	122
1978 I	152	-3	9	-456	144.6	105
II	121	-1	1	-121	141.9	85
III	127	1	1	127	139.1	91
IV	163	3	9	489	136.4	120
1979 I	140	5	25	700	133.6	105
II	100	7	49	700	130.8	76
III	105	9	81	945	128.1	82
IV	158	11	121	1738	125.3	126
	1686	0	572	-788		

Putting these values into the normal eq^{n}s we get,

$1686 = a \times 12 + b \times 0$ ——— (1)

$-788 = a \times 0 + b \times 572$ ——— (2)

From (1), $a = \frac{1686}{12} = 140.5$

From (2), $b = \frac{-788}{572} = -1.38$

Thus the st. line trend eq^n is $y = 140.5 - 1.38x$ with origin at the mid point of quarters II & III of 1978 and unit of $x = \frac{1}{2}$ quarter.

Putting different values of x of different quarters of different years we get the respective trend values which are also given in the same table.

For example: when $x = -11$, then $y = 140.5 - 1.38$...

$= 140.5 + 15.18$

$= 155.68 \simeq 155.$

Now we obtain trend ratio by using the formula:

$$\text{Trend ratio} = \frac{Y}{T} \times 100$$

These are also shown in the same table.

Now in order to find out the seasonal indices by trend ratio method we prepare the following table.

Year/quarter	I	II	III	IV	Total
1977	106	88	93	122	
1978	105	85	91	120	
1979	105	76	82	126	
Total	316	249	266	368	
A.M	105	83	89	123	400
S.I	105	83	89	123	

Here, Grand Average $= 400 \div 4 = 100$

Now, $$S.I = \frac{A.M}{G.A} \times 100 \quad \text{(Ans).}$$

$$= \cancel{400}\ \frac{400}{100} \times 100 = 400 \text{ (Ans).}$$

Suppose we have a series of quarterly production figures (in thousand tons) in an industry for the years 1970 to 1976, and the equation of the linear trend fitted to the annual data is

$$x_t = 107{\cdot}2 + 2{\cdot}97t$$

where t = year − 1973 and x_t = annual production in time period t.

Use this equation to estimate the annual production for the year 1977, and for the year 1971.

Suppose now the quarterly indices of Seasonal variation are :

January-March 125, April - June 105,
July - September 87, October - December 83

[The multiplicative model for the time series is assumed]

Use these indices to estimate the production during the first quarter of 1977.

The equation of the linear trend fitted to the annual data is $X_t = 107.2 + 2.93t$ with origin at the year 1973 and unit of $t = 1$ year.

In 1977, $t = 4$. Thus the estimated annual production in 1977 is

$$X_t = 107.2 + 2.93 \times 4$$
$$= 118.92 \text{ ('000 tons)}$$ Ans

In 1971, $t = -2$. Thus the estimated annual production in 1971 is

$$X_t = 107.2 + 2.93 \times (-2)$$
$$= 101.34 \text{ ('000 tons)}$$ Ans

Again the linear trend equation fitted to quarterly data

$$X_t = \frac{107.2}{4} + \frac{2.93}{4 \times 4}\left(t + \frac{1}{2}\right)$$
$$= 26.8 + 0.183t + 0.09$$
$$= 26.89 + 0.183t$$

Origin : 3rd quarter of 1973 and unit of $t = 1$ quarter

In the first quarter of 1977, $t = 14$

Thus trend value in the 1st quarter of 1977 is

$$X_t = 26.89 + 0.183 \times 14$$
$$= 29.452$$

We are also given,

	I	II	III	IV
Seasonal Index	125	105	87	83
Seasonal effects	1.25	1.05	0.87	0.83

Thus the estimated production during the 1st quarter of 1977 will be $29.452 \times 1.25 = 36.815$ ('000 tons) Ans