LPP

SIMPLEX METHOD

① Solution by Simplex method :-

S.1 Maximise $z = 5x + 6y$

sub. to $x + y \leq 5$

$2x + 3y \leq 12$

$x \geq 0,\ y \geq 0$

→ To solve this problem by simplex method our first task is to convert the inequalities into equalities by introducing slack variables. Since we have two constraints we required two slack variables. Let s_1 and s_2 be the slack variables introduce to convert the ~~inequal~~ inequalities into equalities. Then the problem can be re written as

$$\text{Max. } Z = 5x + 6y + 0.s_1 + 0.s_2$$

Sub to
$$x + y + s_1 + 0.s_2 = 5$$
$$2x + 3y + 0.s_1 + s_2 = 12$$
$$(x \geq 0,\ y \geq 0,\ s_1 \geq 0,\ s_2 \geq 0)$$

The problem is now put into a simplex tableau:-

Simplex tableau-I

C_j			5	6	0	0	
(1)	Basic Variables (2)	Values of the basic variable (3)	x (4)	y (5)	s_1 (6)	s_2 (7)	Replacement Ratio (8)
0	s_1	5	1	1	1	0	$5/1 = 5$
0	s_2	12	2	$\boxed{3}$	0	1	$12/3 = 4$ ←
	Z_j	0	0	0	0	0	
	$C_j - Z_j$	-	5	6 ↑	0	0	

In the 1st programme we include only the slack variables s_1 & s_2 as basic variables thus the 1st solution is $s_1 = 5$; $s_2 = 12$, $x = 0$, $y = 0$. In this solution it is found that the value of the objective fn. is zero. Further from the $C_j - Z_j$ row (Net evaluation row) it is found that there is positive elements which indicates that a better programme can be formulated. The highest positive element in the net evalution row is 6 which lies in y column. Thus y column is the key column & in the next programme y has to be included as 1 of the basic variable. Now dividing the elements of the constant column by non-negative elements of the key column we get the replacement ratios shown in column (8). The lowest ratio appears in the s_2 row & ~~have~~ hence s_2 row is the key row. Thus in the next programme s_2 will be replaced by y.

Here the key number is 3.

Taking s_1 & y as basic variables we construct the second tableau:-

Simplex tableau-II

C_j			5	6	0	0	
(1)	Basic variable (2)	Values of the basic variable (3)	x (4)	y (5)	s_1 (6)	s_2 (7)	replacement ratio
0	s_1	1	$\boxed{1/3}$	0	1	$-1/3$	$1/1/3 = 3$
6	y	4	$2/3$	1	0	$1/3$	$4/2/3 = 6$
	Z_j	24	4	6	0	2	
	$C_j - Z_j$	—	1 ↑	0	0	-2	

In the above table the elements of y row are obtained in dividing the old elements of tableau 1 by key number 3. Again the elements of s_1 row are obtained by the method.

$$\text{New row no.} = \text{Old row no.} - \frac{\text{Corresponding key row number} \times \text{Corresponding key column number}}{\text{key number}}.$$

i.e,
$$5 - \frac{12 \times 1}{3} = 1$$
$$1 - \frac{2 \times 1}{3} = \frac{1}{3}$$
$$1 - \frac{3 \times 1}{3} = 0$$
$$1 - \frac{0 \times 1}{3} = 1$$
$$0 - \frac{1 \times 1}{3} = -\frac{1}{3}$$

Here the key column is the x column & key row is the s_1 row. So, s_1 will be replaced by the variable x. Again as there is one positive element in the net evaluation row, hence better programme can be formulated. Now taking x and y as basic variables we construct the next tableau.

Simplex tableau - III

C_j			5	6	0	0	
(1)	Basic variables (2)	Values of the basic variables (3)	x (4)	y (5)	s_1 (6)	s_2 (7)	Replacement ratio (8)
5	x	3	1	0	3	-1	—
6	y	2	0	1	-2	1	—
	Z_j	27	5	6	3	1	
	$C_j - Z_j$	—	0	0	-3	-1	

In the least table as there is no positive elements in the net evaluation row (all the elements are either 0 or -ve), an optimal solution has been reached and it is not possible to formulate the table for improvement.

Hence the required optimum solution of the given linear programming problem is $x = 3$, $y = 2$ and the corresponding value of $Z = 27$

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Various Types

Q:-2. Maximise $z = 3x_1 + 7x_2 + 6x_3$

sub to $2x_1 + 2x_2 + 2x_3 \leq 8$

$x_1 + x_2 \leq 3$

$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$

→ To solve this problem by simplex method our first task is to convert the inequalities into equalities by introducing slack variable. Since we have two constraints we require two slack variables. Let s_1 and s_2 be the slack variables introduced to ~~convert the inequalities~~ into equalities. Thus the problem can be re-written as

Max. $z = 3x_1 + 7x_2 + 6x_3 + 0.s_1 + 0.s_2$

sub to $2x_1 + 2x_2 + 2x_3 + s_1 + 0.s_2 = 8$

$x_1 + x_2 + 0.x_3 + 0.s_1 + s_2 = 3$

$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0, s_1 \geq 0,\ s_2 \geq 0,$

Now the problem is put in a simplex tableau:-

~~Simplex tableau I~~

Simplex tableau - I

c_j			3	7	6	0	0	0	
(1)	Basic variable (2)	values of the basic variable (3)	x_1 (4)	x_2 (5)	x_3 (6)	s_1 (7)	s_2 (8)		ratio (9)
	Net evaluation Row								

Simplex Tableau - I

C_j			3	7	6	0	0	
(1)	Basic variable (2)	Values of basic variable (3)	x_1 (4)	x_2 (5)	x_3 (6)	s_1 (7)	s_2 8	ratio (9)
0	s_1	8	2	2	2	1	0	8/2 = 4
0	s_2	3	1	$\boxed{1}$	0	0	1	3/1 = 3
	Z_j	0	0	0	0	0	0	
	$C_j - Z_j$	-	3	7	6	0	0	

As there are (+ve) elements in the net evaluation row, better programme can be formulated.

Here the key column is the x_2 column & key row is the s_2 row and key number is 1. So, better programme we replace s_2 variable by x_2 variable in the next ~~part~~ table :-

Simplex tableau-II

c_j			3	7	6	0	0	
(1)	Basic variable (2)	Values of the basic variable (3)	x_1 (4)	x_2 (5)	x_3 (6)	s_1 (7)	s_2 (8)	Replacement ratio (9)
0	s_1	2	0	0	$\boxed{2}$	1	-2	$\frac{2}{2} = 1$ ←
7	x_2	3	1	1	0	0	1	$\frac{3}{0} = \infty$
	z_j	21	7	7	0	0	7	
	$c_j - z_j$	–	-4	0	6 ↑	0	-7	

As there is the positive element in the net evaluation row, hence better programme can be formulated.

Here the key column is the x_3 column & key row is the s_1 row and key no. is 2. So for better programme we replace s_1 variable by x_3 variable in the next table.

Simplex tableau-III

c_j			3	7	6	0	0	
(1)	Basic variables (2)	Value of the basic variable (3)	x_1 (4)	x_2 (5)	x_3 (6)	s_1 (7)	s_2 (8)	Replacement ratio (9)
6	x_3	1	0	0	1	$\frac{1}{2}$	-1	—
7	x_2	3	1	1	0	0	1	—
	z_j	27	7	7	6	3	1	
	$c_j - z_j$	—	-4	0	0	-3	-1	

In the last table as there is +ve element in the net evaluation row (all the elements either 0 or -ve), an optimum solution has been reached & it is not possible to ~~b~~ formulate the ~~table~~ table for improvement.

Hence, the required optimum solution of the given linear programming problem is $x_2 = 3$, $x_3 = 1$ and the corresponding value of $z = 27$.

Here, $z = 27$, $x_2 = 3$, $x_3 = 1$.

Then from the objective fn. we get,

$27 = 3x_1 + 7 \times 3 + 6 \times 1$

$\Rightarrow 27 = 3x_1 + 27$

$\Rightarrow 3x_1 = 0$

$\therefore x_1 = 0$

$\therefore x_1 = 0,\ x_2 = 3,\ x_3 = 1,\ z = 27$ (Ans)

Q. Maximize. $z = 4x + 6y$

s.t $\quad \frac{1}{2}x + y \leq 4$

$2x + y \leq 8$

$4x - 2y \leq 2$

$x \geq 0,\ y \geq 0$

→ To solve this problem by simplex method our 1st task is to convert the inequalities into ~~equah~~ equalities by introducing slack variables. Since we have three constraints we require three slack variables. Let s_1, s_2 and s_3 be the slack variables introduced to convert the inequalities into equalities. Thus the problem can be re- ~~write~~ written as

$$\text{Max. } Z = 4x + 6y + 0.s_1 + 0.s_2 + 0.s_3$$

S.t

$$\frac{1}{2}x + y + s_1 + 0.s_2 + 0.s_3 = 4$$
$$2x + y + 0.s_1 + s_2 + 0.s_3 = 8$$
$$4x - 2y + 0.s_1 + 0.s_2 + s_3 = 2$$

$$x \geq 0, y \geq 0, s_1 \geq 0, s_2 \geq 0, s_3 \geq 0$$

Now the problem is put in a simplex tableau:-

Simplex tableau-I

C_j			4	6	0	0	0	
(1)	Basic variables (2)	Values of basic variables (3)	x (4)	y (5)	s_1 (6)	s_2 (7)	s_3 (8)	Replacement ratio (9)
0	s_1	4	$\frac{1}{2}$	$\boxed{1}$	1	0	0	$\frac{4}{1} = 4$ ←
0	s_2	8	2	1	0	1	0	$\frac{8}{1} = 8$
0	s_3	2	4	-2	0	0	1	-
	Z_j	0	0	0	0	0	0	
	$C_j - Z_j$	—	4	6↑	0	0	0	

In the first table the highest ~~possible~~ positive element in the net evaluation row is 6, so y column is the key column. From the ratio, we find that s_1 is the key row. Here key no. is 1. In the next table we replace s_1 by y.

Simplex tableau - II

c_j			4	6	0	0	0	
(1)	Basic variable (2)	Values of the basic variable (3)	x (4)	y (5)	s_1 (6)	s_2 (7)	s_3 (8)	Replacement ratio (9)
6	y	4	$\frac{1}{2}$	1	1	0	0	$\frac{4}{1/2} = 8$
0	s_2	4	$\frac{3}{2}$	0	-1	1	0	$\frac{4}{3/2} = \frac{8}{3}$
0	s_3	10	$\boxed{5}$	0	2	0	1	$\frac{10}{5} = 2$
	Z_j	24	3	6	6	0	0	
	$C_j - Z_j$	–	1 ↑	0	-6	0	0	

In the second table column x is the key column and row s_3 is the key row. Here the key no. is 5. In the next table we replace s_3 by x.

Simplex tableau - III

C_j			4	6	0	0	0	
(1)	Basic variable (2)	Values of the basic variable (3)	x (4)	y (5)	s_1 (6)	s_2 (7)	s_3 (8)	Replacement ratio (9)
6	y	3	0	1	4/5	0	-1/10	
0	s_2	1	0	0	-8/5	1	-3/10	
4	x	2	1	0	2/5	0	1/5	
	Z_j	26	4	6	32/5	0	1/5	
	$C_j - Z_j$	–	0	0	-32/5	0	-1/5	

In the last table all the elements in the net evaluation row are either 0 or (-ve). So; any better programme can't be formulated further. Thus the optimum solution of the problem is

$x = 2, y = 3$ and $z = 26$ (Ans.)

Simplex method for solving a minimisation problem:-

Q. Minimise $W = 5u + 12v$

s.t

$$u + 2v \geq 5$$
$$u + 3v \geq 6$$
$$u \geq 0, \ v \geq 0$$

→ To solve this problem our first task is to convert the inequalities into equalities by substructing slack variables and adding artificial slack variables. Let S_1 and S_2 be the slack variables and let A_1 and A_2 be the artificial slack variables. So, we re-write the problem as

Min. $W = 5u + 12v + 0.S_1 + 0.S_2 + M.A + M.A_2$

s.t

$$u + 2v - S_1 + 0.S_2 + A_1 + 0.A_2 = 5$$
$$u + 3v + 0.S_1 - S_2 + 0.A_1 + A_2 = 6$$
$$(u, v, S_1, S_2, A_1, A_2) \geq 0$$

Now we construct the simplex tableau-I

P.T.O →

c_j			5	12	0	0	M	M	
(1)	Basic Variable (2)	Values of the basic variable (3)	u (4)	v (5)	s_1 (6)	s_2 (7)	A_1 (8)	A_2 (9)	Replacement ratio (10)
M	A_1	5	1	2	-1	0	1	0	$\frac{5}{2} = 2.5$
← M	A_2	6	1	$\boxed{3}$	0	-1	0	1	$\frac{6}{3} = 2$
	Z_j	11 M	2M	5M	-M	-M	M	M	
	$C_j - Z_j$	–	5-2M	12-5M ↑	M	M	0	0	

~~In the first table~~ the ~~highest~~ ~~possible~~ negative ~~element~~ ~~in the~~.

In the above table there are two negative elements in the net evaluation row such as (5-2M) and (12-5M). Between the two the largest negative element is (12-5M). So, column v is the key column. From replacement ratio we get the row A_2 is the key row. The key no. is 3.

In the next table we replace A_2 by v.

simplex tableau - II

C_j			5	12	0	0	M	M	
(1)	Basic variables (2)	Values of the basic variable (3)	u (4)	v (5)	s_1 (6)	s_2 (7)	A_1 (8)	A_2 (9)	Replacement (10)
M	A_1	1	$\frac{1}{3}$	0	-1	2/3	1	-2/3	$\frac{1}{2/3} = 3/2$ ←
12	v	2	$\frac{1}{3}$	1	0	-1/3	0	1/3	–
	Z_j	M + 24	$\frac{M}{3}+4$	12	-M	$\frac{2M}{3}-4$	M	$-\frac{2M}{3}+4$	
	$C_j - Z_j$	–	$-\frac{M}{3}+1$	0	M	$-\frac{2M}{3}+4$ ↑	0	$\frac{5M}{3}-4$	

Here in the net evaluation row we have two (-ve) elements such as $-\frac{M}{3}+1$ and $-\frac{2M}{3}+4$. Between the two $-\frac{2M}{3}+4$ is the Largest (-ve) element. So, S_2 column is the key column and from the ratio A_1 row is the key row. Here the key no. is $\frac{2}{3}$. Now we replace A_1 by S_2 in the next table.

Simplex tableau-III

C_j			5	12	0	0	M	M	
(1)	Basic variable (2)	Value of the basic variable (3)	u (4)	v (5)	S_1 (6)	S_2 (7)	A_1 (8)	A_2 (9)	Replacement ratio (10)
0	S_2	3/2	$\boxed{1/2}$	0	-3/2	1	3/2	-1	$\frac{3/2}{1/2} = 3$
12	v	5/2	1/2	1	-1/2	0	1/2	0	$\frac{5}{2}/\frac{1}{2} = 5$
	Z_j	30	6	12	-6	0	6	0	
	$C_j - Z_j$	—	-1 ↑	0	6	0	M-6	M	

In the table there is only one (-ve) element the ~~nearest~~ net evaluation row which is -1. So column u is the key column. Again from the ratio, the key row is the S_2 row. So, we replace S_2 by u in the next table.

Simplex tableau-IV

C_j			5	12	0	0	M	M	
(1)	Basic variables (2)	Value of the basic variable (3)	u (4)	v (5)	S_1 (6)	S_2 (7)	A_1 (8)	A_2 (9)	Replacement ratio (10)
5	u	3	1	0	-3	2	3	-2	
12	v	1	0	1	1	-1	-1	1	
	Z_j	27	5	12	-3	-2	3	2	
	$C_j - Z_j$	—	0	0	3	2	M-3	M-2	

It is found ~~for~~ from the table that all the elements in the net evaluation row are (+ve). So, further programme can't be formulated. Here the optimum solution is $u = 3, v = 1$ and all other variables equal to zero. Here $w = 27$

Q-2 Minimise $Z = 4u + 8v + 2w$

$$\text{s.t} \quad \frac{1}{2}u + 2v + 4w \geq 4$$
$$u + v - 2w \geq 6$$
$$u \geq 0,\ v \geq 0,\ w \geq 0.$$

→ To solve this problem our ~~first task~~ is to convert the inequalities into equalities by substructing slack variables and adding artificial slack variables. Let s_1 and s_2 be the slack variables and let A_1 and A_2 be the artificial slack variables. So, we rewrite the problem as,

$$\text{Min } Z = 4u + 8v + 2w + 0.s_1 + 0.s_2 + M.A_1 + M.A_2$$

$$\text{s.t} \quad \frac{1}{2}u + 2v + 4w - s_1 + 0.s_2 + A_1 + 0.A_2 = 4$$
$$u + v - 2w + 0.s_1 - s_2 + 0.A_1 + A_2 = 6$$
$$(u, v, w, s_1, s_2, A_1, A_2) \geq 0$$

Now we construct simplex tableau-I.

Simplex Tableau - I

C_j			4	8	2	0	0	M	M	
(1)	Basic variables (2)	Values of the basic variable (3)	u (4)	v (5)	w (6)	s_1 (7)	s_2 (8)	A_1 (9)	A_2 (10)	Replacement ratio (11)
M	A_1	4	$\frac{1}{2}$	$\boxed{2}$	4	−1	0	1	0	$\frac{4}{2} = 2$
M	A_2	6	1	1	−2	0	−1	0	1	$\frac{6}{1} = 6$
	Z_j	10M	$\frac{3}{2}M$	3M	2M	−M	−M	M	M	
	$C_j - Z_j$	—	$4 - \frac{3M}{2}$	8 − 3M ↑	2−2M	M	M	0	0	

In the above table there are three (−ve) elements in the net evaluation row such as $4 - \frac{3M}{2}$, 8 − 3M and 2 − 2M. Between the three largest (−ve) element is 8 − 3M. So, column v is the key ~~row the~~ column. From ratio we get row A_1 is the key row. The key no. is 2. In the next table we replace A_1 by v.

Simplex tableau-II

C_j			4	8	2	0	0	M	M	
(1)	Basic variables (2)	Value of the basic variables (3)	u (4)	v (5)	w (6)	s_1 (7)	s_2 (8)	A_1 (9)	A_2 (10)	ratio (11)
8	v	2	$\frac{1}{4}$	1	2	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	
M	A_2	4	3/4	0	−4	$\frac{1}{2}$	−1	$-\frac{1}{2}$	1	
	Z_j	16 + 4M	$2 + \frac{3M}{4}$	8	16 − 4M	$-4 + \frac{M}{2}$	−M	$4 - \frac{M}{2}$	M	
	$C_j - Z_j$	—	$2 - \frac{3M}{4}$	0	−14 + 4M	$4 - \frac{M}{2}$	M	$\frac{-4 + 3M}{2}$	0	

Here the net evaluation row we have two (-ve) elements such as $2-\frac{3M}{4}$ and $4-\frac{M}{2}$. Between the u column is the key column and from the ratio A_2 row is the key row. Here the key number is $\frac{3}{4}$. Now we replace A_2 by u in the next table.

Simplex-III tableau

C_j			4	8	2	0	0	M	M	
(1)	Basic variable (2)	Values of the basic variable (3)	u (4)	v (5)	w (6)	s_1 (7)	s_2 (8)	A_1 (9)	A_2 (10)	ratio (11)
8	v	2/3	0	1	$\frac{10}{3}$	$-2/3$	$\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$2/3 / \frac{16}{3} = \frac{1}{5}$
4	u	16/3	1	0	$-\frac{16}{3}$	2/3	$-\frac{4}{3}$	$-\frac{2}{3}$	4/3	–
	Z_j	80/3	4	8	$\frac{16}{3}$	$-8/3$	$-8/3$	$8/3$	8/3	
	$C_j - Z_j$	–	0	0	$-\frac{10}{3}$	8/3	8/3	$M-\frac{8}{3}$	$M-\frac{8}{3}$	

Here the net evaluation row we have one (-ve) element such as $-\frac{10}{3}$. So, column w is the key column and from ratio v is the key row. Here the key no. is $\frac{10}{3}$. Now we replace v by w in the next table.

Simplex tableau-5

C_j			4	8	2	0	0	M	M	
(1)	Basic variables (2)	Values of the basic variables (3)	u (4)	v (5)	w (6)	s_1 (7)	s_2	A_1	A_2	ratio
2	w	$\frac{1}{5}$	0	$\frac{3}{10}$	1	$-\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{5}$	$-\frac{1}{10}$	—
4		$32/5$	1	$\frac{16}{10}$	0	$-\frac{2}{5}$	$-\frac{4}{5}$	$\frac{2}{5}$	$\frac{4}{5}$	—
	Z_j	26	4	7	2	-2	-3	2	3	
	$C_j - Z_j$	—	0	1	0	2	3	M-2	M-3	

It is found from the table that all the elements in the net evaluation row are possitive. So, further programming can't be formulated. Here $w = \frac{1}{5}$, $u = \frac{32}{5}$ and $Z = 26$.

From objective fn., we get

$$26 = 4 \times \frac{32}{5} + 8 \times v + 2 \times \frac{1}{5}$$

$$\text{or, } 26 = \frac{128}{5} + 8v + \frac{2}{5}$$

$$\text{or, } \cancel{26}\ 8v = 26 - \frac{128}{5} - \frac{2}{5}$$

$$= \frac{130 - 128 - 2}{5}$$

$$= 0.$$

$\therefore v = 0$

Thus, the optimum solution is $u = \frac{32}{5}$, $v = 0$, $w = \frac{1}{5}$ and, $Z = 26$. (Ans)

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Duality of linear programming:-

Every mathematical linear programming problem is intimately (ঘনিষ্ঠভাবে) related to another linear programming problem called its dual. For purposes of ~~identified~~ identification the original problem is called the primal problem. The relationship between the primal problem and the dual problem can be summarised as follows:-

i) The dual has as many variables as there are contraints in the original problem.

ii) The dual has as many constraints as there are variables in the original problem.

(iii) The dual of a maximisation problem is a minimisation problem and vice-versa.

(iv) The co-efficient of the objective fn. of the original problem appear as the constraint terms of the constraints of the dual, and the constraint terms of the ~~obj~~ original constraints are the co-efficients of the objective fn. of the dual.

(v) The co-efficient of a single variable in the original ~~constraints~~ constraints become the co-efficient of a single ~~co~~ constraint in the dual. Stated visually, each column of co-efficient in the constraints of the original problem becomes a row of co-efficients in the dual.

(vi) The sense of the inequalities in the dual is the reverse of the sence of the inequalities in the original problem, except that the inequalities restricting the variables to be non-negative have the same ~~sense~~ sense in the primal and the dual.

To clearify the relation between the primal and the dual let us take one example. Suppose our ~~primal~~ problem is to

$$\text{Max. } Z = c_1 x_1 + c_2 x_2$$

s.t

$$a_{11} x_1 + a_{12} x_2 \leq b_1$$

$$a_{21} x_1 + a_{22} x_2 \leq b_2$$

$$a_{31} x_1 + a_{32} x_2 \leq b_3$$

$$(x_1 \geq 0,\ x_2 \geq 0)$$

Since the original problem has 3 constraints, the dual problem will have 3 variables. Let y_1, y_2, y_3 be the dual variables. Again, since the original problem has two variables the dual problem will have two constraints. Again since the original is a maximisation problem the dual problem will be a minimisation problem. Again since the constraints in the original problems are of "less than equal to" type, the constraints in the dual problem will be "greater than equal to" type. Thus the dual problem then be as follows:-

$$\text{Min. } w = b_1 y_1 + b_2 y_2 + b_3 y_3$$

s.t

$$a_{11} y_1 + a_{21} y_2 + a_{31} y_3 \geq c_1$$

$$a_{12} y_1 + a_{22} y_2 + a_{32} y_3 \geq c_2$$

$$y_1 \geq 0,\ y_2 \geq 0,\ y_3 \geq 0$$

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Importance of the duality:-

The knowledge of the dual is important for two main reasons. First, the dual variable have economic interpretation. The values of the dual variables may be useful in taking managerial decisions. Second, the solution of the L.P.P may be easier to obtain through the dual than through the primal problem. This is particularly true for those primal problems where there are more slack variables than ordinary variables. Suppose we are considering a maximisation problem involving two variables x and y and 5 constraints. In this case the intial simplex table of this problem will have 5 rows and to transform it to the next table we have to make a lot of calculations but if we form the dual of this problem it will have ony two artificial slack variables and the initial simplex table will have only two rows. Here transformation from one table to another become relatively easy. Thus the original maximisation problem can be solved more easily via its dual

Q.1

Consider 1 numerical example as,

Maximise $Z = 5x + 6y$

Sub to. $x + y \leq 5$

$2x + 3y \leq 12$

→ In the original problem we have two constraints. Thus there will be two variables in the dual problem. Let u & v be the dual variable. Then the dual problem can be written as,

Minimise $W = 5u + 12v$

S.t $u + 2v \geq 5$

$u + 3v \geq 6$

$u \geq 0, v \geq 0$ (Ans.)

Q.2

Minimise $W = 4u + 8v + 2w$

s.t. $\frac{1}{2}u + 2v + 4w \geq 4$

$u + v - 2w \geq 6$

$u \geq 0, v \geq 0, w \geq 0$

→ In the original problem we have two constraints. Thus there will be two variables in the dual problem & 3 constraints. Let x & y be the dual variables. Then the dual problem can be written as.

Maximise $Z = 4x + 6y$

S.t $\frac{1}{2}x + y \leq 4$

$2x + y \leq 8$

$4x - 2y \leq 2$

$x \geq 0, y \geq 0$ (Ans)

** In example 3 the elements in the net evaluation row under column s_1, s_2 and s_3 are $-\frac{32}{5}$, 0, $-\frac{1}{5}$. Hence the solution of the 3 dual variables will be $u = \frac{32}{5}$, $v = 0$ and $w = \frac{1}{5}$

Similarly in the dual problem the elements in the net evaluation row under column s_1, s_2 are 2 & 3 respectively. They represents the values of x & y in the original problem. Thus the solution $x = 2$ & $y = 3$ of the original problem can be determined from the dual problem.

Degeneracy :-

In the simplex method the replacement ratios are essential to identify the key row. In this case we may face the following situations :-

(1) There may be only one non-negative ratio which is the smallest. In this case, the simplex method can be applied to solve the problem.

(2) The non-negative entries in the key column may be such that either the minimum replace-ment ratio is 0 or there is a 'tie' between two or more minimum replacement ratios. In such a case the problem is to be degenerate.

(3) It may happen that all the entires in the key column are either 0 or (-ve). In this LPP ~~does~~ not have any solution.

In case of degeneracy, that variable should be removed ~~first~~ which found first in a table of random numbers. Another rule is denote all the variables by uniform notations like x_1, x_2, x_3 etc. and select that row as the key row which has the smallest subscript.

By- Dr. R. K. GHOSH.