THE JACOBI-ITERATION METHOD

- ***THEORY :***

Let the system of linear equations, given by,

$$a_{11}x_1+ a_{12}x_2+.....+a_{1n}x_n=b_1$$
$$a_{21}x_1+a_{22}x_2+.....+a_{2n}x_n=b_2$$
$$.....$$
$$a_{n1}x_1+a_{n2}x_2+.....+a_{nn}x_n=b_n$$

has unique solutions.

The coefficient matrix A has no non-zero diagonal elements,i.e, $a_{11},a_{22},.....,a_{nn}$ are non-zeros.

To solve the 1st equation for x_1, the 2nd equation for x_2 and so on, we have to rewritten equations as :

$$x_1=(1/a_{11})*(b_1-a_{12}x_2-a_{13}x_3-....-a_{1n}x_n)$$
$$x_2=(1/a_{22})*(b_2-a_{21}x_2-a_{23}x_3-....-a_{2n}x_n)$$
$$.....\qquad (2)$$
$$x_n=(1/a_{nn})*(b_n-a_{n1}x_2-a_{n2}x_3-....-a_{n.n-1}x_{n-1})$$

Let make an initial guess of the solution $x^{(0)}=(x_1^{(0)},x_2^{(0)},...,x_n^{(0)})$ is equal to zero. Substitute these values to equations (2) for the first approximation of $x_i^{(1)}$. This accomplishes one iteration. Then use these values as guess values to get the 2nd order approximation. This accomplishes 2nd iteration and so on. Similarly, let after kth iteration we found the required accuracy in the solution. Then the kth iteration values of the roots $x_i^{(k)}$s are the solutions of the given system of equations. Here i runs from 1 to n.

This method is known as **Jacobi-Iteration method**.

- ***ALGORITHM :***

1. Declare variables
2. Read the order and the coefficients of the system of equations from data file
3. Enter the conditions for the validation of this method
4. Set the initial values to zero
5. Set x(i)=0 and let xp =0
6. do I=1,n
7. x(i)=xp(i)
8. y(i)=x(i)
9. sum=b(i)
10. if i=j then !do nothing

11. else
12. sum=sum-a(i,j)*x(j)
13. endif
14. enddo
15. xp(i)=sum/a(i,j)
16. if xp(i)-y(i)<0.0001 then
17. set =1
18. else
19. set=0
20. sum1=0
21. do i=1,n
22. sum1=sum1+i
23. if sum1=n then goto 25
24. else goto 5
25. endif
26. Read theoretical values of the roots from data file nad calculate the percentage error
27. Write roots xp(i), number of iterations and percentage error

- ***INPUT DATA FILE FOR ORDER AND COEFFICIENTS OF THE SYSTEM AND THE THEORETICAL ANSWER OF THE EQUATIONS :***

```
2
3.000000        1.000000        5.000000
1.000000       -3.000000        5.000000
2.000000       -1.000000
```

- ***FORTRAN CODE :***

```
!   solution of linear equations by Jacobi-Iteration method

    real sum,sum1,count,prod,perc,perc1
    integer i,j,n
    dimension A(10,10),B(10),x(10),y(10),set(10),xp(10),z(10)
    open (unit=1,file='datame.dat')
    read (1,*) n
    do i=1,n
    read(1,*) (A(i,j),j=1,n),B(i)
```

```
      enddo
      prod=1
      do i=1,n
      do j=1,n
      if (i.eq.j) then
      else
      prod=prod*A(i,j)
      endif
      if (A(i,i).gt.prod) then
      goto 30
      else
      write(*,*) "We can't solve the set of equations by this method."
      goto 40
      endif
      enddo
      enddo

      write(*,*) "Set the values of the unknowns to zero."
30    do i=1,n
      x(i)=0
      xp(i)=0
      enddo

      count=0
10    do i=1,n
      x(i)=xp(i)
      y(i)=x(i)
      sum=B(i)
      do j=1,n
      if (i.eq.j) then
      else
      sum=sum-A(i,j)*x(j)
      endif
      enddo
      xp(i)=sum/A(i,i)
      if (abs(xp(i)-y(i)).lt.0.000001) then
      set(i)=1
      else
      set(i)=0
      endif
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```
      enddo
      sum1=0
      do i=1,n
      sum1=sum1+set(i)
      enddo
      if (sum1.eq.n) then
      goto 20
      else
      count=count+1
      goto 10
      endif
      end file 1
20    write(*,*) "The roots of the unknowns of the equations are :"
      write(*,*) (xp(i),i=1,n)
      write(*,*) "Number of iterations are :",count

      read (1,*) (z(i),i=1,n)
      perc=0
      do i=1,n
      perc=perc+(((z(i)-xp(i))/z(i))*100)
      enddo
      perc1=perc/n
      write(*,*) "The percentage error in this calculation is :",perc1

40    stop
      end
```

- ***OUTPUT :***

```
The roots of the unknowns of the equations are :
     2.000000
   -9.999998E-01
Number of iterations are :          21.000000
The percentage error in this calculation is :      8.940697E-06
Stop - Program terminated.
```