GAUSS ELIMINATION METHOD

- ***THEORY :*** Gauss elimination is a method for solving matrix equations of the form,

  Ax = b (1)

To perform Gaussian elimination starting with the system of equations

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1k} \\ a_{21} & a_{22} & \cdots & a_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ a_{k1} & a_{k2} & \cdots & a_{kk} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_k \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_k \end{bmatrix}, \quad (2)$$

Compose the “argument matrix equation”

$$\left[\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1k} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2k} & b_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{k1} & a_{k2} & \cdots & a_{kk} & b_k \end{array}\right] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_k \end{bmatrix}. \quad (3)$$

Here the column vector in the variables x is carried along for labeling the matrix rows. Now,perform elementary row operations to put the argument matrix into the upper triangular form

$$\left[\begin{array}{cccc|c} a'_{11} & a'_{12} & \cdots & a'_{1k} & b'_1 \\ 0 & a'_{22} & \cdots & a'_{2k} & b'_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & a'_{kk} & b'_k \end{array}\right]. \quad (4)$$

Solve the equation of the k^{th} row for x_k ,then substitute back into the equation of the $(k+1)^{st}$ row to obtain a solution for x_{k-1}, etc., according to the formula

$$x_i = \frac{1}{a'_{ii}} \left(b'_i - \sum_{j=i+1}^{k} a'_{ij} x_j \right). \quad (5)$$

This is the back substitution process and the whole method is the known as **Gauss elimination method**.

- ***ALGORITHM :-***
    1. Declare variables
    2. Read the order of the matrix
    3. Read the coefficients of the matrix
    4. do k=1,n-1
    5. do i=k+1,n
    6. do j=k+1,n+1
    7. A(i,j)=A(i,j)-A(i,k)/A(k,k)*A(k,j)
    8. enddo
    9. enddo
    10. enddo
    11. x(n)=A(n,n+1)/A(n,n)
    12. do k=n-1,1
    13. Sum=0
    14. do j=k+1,n
    15. Sum=Sum+A(k,j)*x(j)
    16. enddo
    17. x(k)=(1/A(k,k))*(A(k,n+1)-Sum)
    18. enddo
    19. Write the solutions x(k)

- ***FORTRAN CODE:***

```
!    solution of system of equations by Gauss Elimination method
     real sum,A(10,10),x(10)
     integer n,i,j,k

     write(*,*) "Enter the order of the system :"
     read(*,*) n
     write(*,*) "Enter the coefficients of the system of equations."
     do i=1,n
     read(*,*) (A(i,j),j=1,n+1)
     enddo

     do k=1,n-1
     do i=k+1,n
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    do j=k+1,n+1
    A(i,j)=A(i,j)-A(i,k)/A(k,k)*A(k,j)
    enddo
    enddo
    enddo

    x(n)=A(n,n+1)/A(n,n)

    do k=n-1,1,-1
    sum=0
    do j=k+1,n
    sum=sum+A(k,j)*x(j)
    enddo
    x(k)=1/A(k,k)*(A(k,n+1)-sum)
    enddo
    write(*,*) "The solutions of the equations are :", (x(i),i=1,n)
    stop
    end
```

- ***OUTPUT :***

```
    Enter the order of the system :
    3
    Enter the coefficients of the system of equations.
    2 1 3 1
    4 4 7 1
    2 5 9 3
    The solutions of the equations are :  -5.000000E-01
-1.000000
                1.000000
    Stop - Program terminated.
```