GAUSS-SEIDAL METHOD

- ***THEORY :***

Gauss-Seidal method is a modification of the Jacobi-Iteration method. Let the system of linear equations, given by,

$$a_{11}x_1+ a_{12}x_2+.......+a_{1n}x_n=b_1$$
$$a_{21}x_1+a_{22}x_2+........+a_{2n}x_n=b_2$$
$$................$$
$$a_{n1}x_1+a_{n2}x_2+........+a_{nn}x_n=b_n$$

has unique solutions.

The coefficient matrix A has no non-zero diagonal elements,i.e, $a_{11},a_{22},.......,a_{nn}$ are non-zeros.

To solve the 1^{st} equation for $x_{1,}$the 2^{nd} equation for x_2 and so on, we have to rewritten equations as :

$$x_1=(1/a_{11})*(b_1-a_{12}x_2-a_{13}x_3-....-a_{1n}x_n)$$
$$x_2=(1/a_{22})*(b_2-a_{21}x_2-a_{23}x_3-....-a_{2n}x_n)$$
$$............... \qquad (2)$$
$$x_n=(1/a_{nn})*(b_n-a_{n1}x_2-a_{n2}x_3-....-a_{n.n-1}x_{n-1})$$

In this method we put $x_i^{(0)}=0$ in the first equation to find the new value of x_1 ,then we find the value of x_2 using the new value of x_1 and the other guess values. Then for next one we replace the value of x_2 to new value and so on, simultaneously. Let after k^{th} iteration we found the required accuracy in the solution. Then the k^{th} iterative values of $x_i^{(k)}$ are the solutions of the given system of equations. Here i runs from 1 to n.

This is the theory of the **Gauss-Seidal method**.

- ***ALGORITHM :-***

1. Declare variables
2. Read order and the coefficients of the system of equations
3. Set initial values of the unknowns to zero
4. x(i)=0
5. do =1,n
6. y(i)=x(i)
7. sum=B(i)
8. do j=1,n
9. if i=j , then
   ! Do nothing
10. else

11. sum=sum-A(i,i)*x(j)
12. endif
13. enddo
14. x(i)=sum/A(i,i)
15. If abs(x(i)-y(i))< 0.00001 then
16. Set(i)=1
17. else
18. Set(i)=0
19. endif
20. enddo
21. Sum1=0
22. do i=1,n
23. Sum1=Sum1+Set(i)
24. enddo
25. if Sum1=n then
26. goto (30)
27. else
28. goto (5)
29. endif
30. Write the roots of the equations, x(i)

- ***<u>FORTRAN CODE :</u>***

```
!    solution of linear equations by Gauss-Seidal iteration method

     real sum,sum1,count
     integer i,j
     dimension A(10,10),B(10),x(10),y(10),set(10)
     write(*,*) "Enter the order of the system :"
     read (*,*) n

     write(*,*) "Enter the coefficients of the system of equations."
     do i=1,n
     read(*,*) (A(i,j),j=1,n),B(i)
     enddo

     write(*,*) "Set the values of the unknowns to zero."
     do i=1,n
     x(i)=0
     enddo
```

```
      count=0
10    do i=1,n
      y(i)=x(i)
      sum=B(i)
      do j=1,n
      if (i.eq.j) then
      else
      sum=sum-A(i,j)*x(j)
      endif
      enddo
      x(i)=sum/A(i,i)
      if (abs(x(i)-y(i)).lt.0.000001) then
      set(i)=1
      else
      set(i)=0
      endif
      enddo
      sum1=0
      do i=1,n
      sum1=sum1+set(i)
      enddo
      if (sum1.eq.n) then
      goto 20
      else
      count=count+1
      goto 10
      endif
20    write(*,*) "The roots of the unknowns of the equations are :"
      write(*,*)  (x(i),i=1,n)
      write(*,*) "Number of iterations are :",count
      stop
      end
```

- ***OUTPUT :***

```
Enter the order of the system :
3
Enter the coefficients of the system of equations.
2 1 3 1
4 4 7 1
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```
2 5 9 3
Set the values of the unknowns to zero.
The roots of the unknowns of the equations are :    -4.999982E-01
   -9.999981E-01       9.999985E-01
Number of iterations are :          35.000000
Stop - Program terminated.
```