# Charge Conjugation.

Concider an operator (charge Conjugation) C, which Converts particle to it corresponding antiparticle.

So if $|\psi\rangle$ represents particle state and $|\bar{\psi}\rangle$ its anti-particle then

$$C|\psi\rangle = |\bar{\psi}\rangle$$

Similarly, the reverse is also true.

$$C|\bar{\psi}\rangle = |\psi\rangle$$

Thus,
$$C^2|\psi\rangle = C\,C|\psi\rangle = C|\bar{\psi}\rangle = |\bar{\psi}\rangle.$$

thus this follows the ~~given~~ eigenvalue of charge conjugation to be $\pm 1$ just like Parity operator.

Since it converts part. to its antipart. & vice versa, it reverses the sign of all quantum no. (changing the sign of magnetic moment).

Consider a proton $|P\rangle$ having positive charge $q$

$$\therefore \; Q|P\rangle = q|P\rangle. \quad \text{with baryon number } B=+1$$

So on operating C, we have $C|P\rangle = |\bar{P}\rangle$ which is an antiparticle of proton with -ve charge.

$$\therefore \; Q|\bar{P}\rangle = -q|\bar{P}\rangle.$$

$$\left[ Q = \text{charge operator.} = \int d^3x \, J^0 \quad \text{where } J^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \delta\phi \right]$$

Thus the Baryon number has been changed to $B = -1$.

Here it is to be noted that $|P\rangle$ is not an eigen state of C as $C|P\rangle = |\bar{P}\rangle$ and $|\bar{P}\rangle$ is completely a different state with diff. quantum number. However $|P\rangle$ is an eigenstate of $C^2$ operator.

So for any state to be eigen state of C should have zero charge. [$\because$ in anti part. has opp sign of part]

Thus any state with zero charge is an eigen state of C. for example neutral pion $\pi^0$

$$\therefore \; C|\pi^0\rangle = \alpha|\pi^0\rangle. \qquad \alpha = \text{eigen Value}$$

So eigen value ~~state~~ should be $\pm 1$.

# Charge conjugation properties of Photon

One can conclude that C operator when operated to the eigen state defining the charged particle, it actually changes the ~~Singn~~ Sign of charge density. So we can say

$$C J C^{-1} = -J$$

Now, the interaction part of electromagnetic lagrangian can be used to determine the charge conjugation properties of photon. So we will see the action of C on $J_\mu A^\mu$,

$$\therefore \quad C J_\mu A^\mu C^{-1} = C J_\mu C^{-1} C A^\mu C^{-1}$$

$$= - J_\mu C A^\mu C^{-1}$$

So to be invarient above eq$^n$ should follow. $\underline{C A^\mu C^{-1} = -A^\mu}$

$$\therefore \quad C J_\mu A^\mu C^{-1} = + J_\mu A^\mu .$$

This means since $A^\mu$ rep. ele.mag. vector pot. the eigen value of charge conjugation of photon is $\alpha = -1$.

So for n ~~sub~~ number of photon it is $(-1)^n$.

Since $\pi^0$ disintegrate into two photon

$$\pi^0 = \gamma + \gamma .$$

$$\therefore \text{ eigen value of } \pi^0 = (-1)^n = (-1)^2 \quad \because n = 2$$

$$= +1$$

This proves that charge conjugation proceeds similar to Parity. So one can say

1. C is conserved in strong & elec. mg. interaction
2. C is not Conserved in Weak Interaction

# C.P Violation

Individual violation of C & P in weak interaction suggest the conservation of the combined C&P it is ~~also~~ almost true but the there lies slight violation in this too from the decay of neutral K mesons.

neutral K mesons $[|K^0\rangle]$ is a linear combination of states with its antiparticle. which is due to spontaneous transition of its state to antiparticle and vice verra.

ie. $$|K^0\rangle \leftrightarrow |\bar{K}^0\rangle$$

$K^0$ mesons being a pseudoscaler $0^-$, its parity is -ve.

$$P|K^0\rangle = -|K^0\rangle$$

$$P|\bar{K}^0\rangle = -|\bar{K}^0\rangle$$

So charge conjugation as already known is

$$C|K^0\rangle = |\bar{K}^0\rangle$$

$$C|\bar{K}^0\rangle = |K^0\rangle$$

on combination,

$$CP|K^0\rangle = -C|K^0\rangle = -|\bar{K}^0\rangle$$

$$CP|\bar{K}^0\rangle = -C|\bar{K}^0\rangle = -|K^0\rangle$$

thus both $|\bar{K}^0\rangle$ and $|K^0\rangle$ is not the eigen state of CP. To observe the violation of CP, we construct the eigen state of CP which is a combination of two state $|K^0\rangle$ & $|\bar{K}^0\rangle$.

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$$|K_1\rangle = \frac{|K^0\rangle - |\bar{K}^0\rangle}{\sqrt{2}} \quad ; \quad |K_2\rangle = \frac{|K^0\rangle + |\bar{K}^0\rangle}{\sqrt{2}}.$$

This new state $|K_1\rangle$ and $|K_2\rangle$ canbe visuallize as the ation of $|K^0\rangle$ & $|\bar{K}^0\rangle$ is ~~two~~ vector space by an angle $\theta = \frac{\pi}{4}$

ie. $$\begin{pmatrix} |K_1\rangle \\ |K_2\rangle \end{pmatrix} = R(\theta) \begin{pmatrix} |K_0\rangle \\ |\bar{K}_0\rangle \end{pmatrix}$$ where $\theta = -\frac{\pi}{4}$.

R in two dimension.

$$= \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta. \end{pmatrix}$$

So. we have,

$$CP\,|K_1\rangle = +|K_1\rangle$$

$$CP\,|K_2\rangle = -|K_2\rangle$$

this state canbe created in laboratory. and it turns out that both decays into $\pi^0$ mesons. So if CP is to be conserved, then each of these states will decay into a state with same CP value.

ie $|K_1\rangle$ decays into a state with $CP = +1$

& $|K_2\rangle$ " " " " $CP = -1$.

The neutral K mesons disintegrate with two $\pi$ mesons & $3\pi$ mesons. So. $2\pi$ meson $\rightarrow C = 1 \quad P = +1 \Rightarrow CP = 1$

$3\pi$ " $\rightarrow C = +1, P = -1 \Rightarrow CP = -1$

This means $|K_1\rangle$ decays ~~only~~ ~~by~~ into $2\pi$ mesons.

where as $|K_2\rangle$ " " $3\pi$ "

But this is not what is observed experimentally. Small fraction of $|K_2\rangle$ decays to $2\pi$ mesons. ie- $CP = -1$ has been transist to $CP = +1$ state.

So, it is observed that long lived, K mesons is,

$$\boxed{|K_L\rangle = \frac{|K_2\rangle + \epsilon|K_1\rangle}{\sqrt{1+|\epsilon|^2}}}$$

Where $\epsilon$ is the measure of the amount of violation of CP conservation.

Expermentally it value is $\boxed{\epsilon = 2.3 \times 10^{-3}}$

$\epsilon$ is small but is not zero. hence ie CP is voileted in this decay.

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## CPT theorem

To restore the CP invariance, we introduce one more Symmetry, time reversal. This is anoter discrete trasformation on states, turning a state $|\psi\rangle$ into another state $|\psi'\rangle$ that evolves with time flow in the -ve direction. In this operation all momentum changes its sign. So.

$$\boxed{T|\psi\rangle = |\psi'\rangle.}$$

~~thim~~ time reversal operator is antiunitary and antilinear. When we say it is antilinear, it means,

$$\boxed{T(\alpha|\psi\rangle + \beta|\phi\rangle) = \alpha^*|\psi'\rangle + \beta^*|\phi'\rangle}$$

Similarly when we say it is anti unitary it mens that it does not preserve inner product (as does by unitary)

ie $\boxed{\langle T\phi|T\psi\rangle = \langle\phi|\psi\rangle^*}$

Thus one can say that time-reversal operator is a Combination of unitary operator and the operator say K which convers states to its complex conjugate.

ie. $T = UK$

where K is such that, $K|\psi\rangle = |\psi\rangle^*$

So if T is such that $[T, H] = 0$ & if $|\psi\rangle$ is a sol$^n$ of Schrodinger eq$^n$ then $T|\psi\rangle$ is also the sol$^n$ of schrödinger eq$^n$ with $t = -t$. thus this suggest the name time-reversal.

If the laws of physics remains invariant under time-reversal ie. $t = -t$, then this is the symmetry of the system.

CPT theorm

The CPT theorem considers 3 symmetries, C, P & T taken together as CPT.

According to this theorm,

If charge conjugation, parity reversal, and time-reversal are taken together we have an exact symmetry and the ~~ss~~ laws of physics remains invariant.

This means if matter is replaced by antimatter (charge conjugation), momentum is reversed with spatial inversion and time is reversed, the result ~~will~~ would be a universe indistinguisible from one we live in.

For CPT theorm to be valid, all three symmetry should be valid or if one or more symmetries are violeted then other symmetry must be violated to cancel the 1st violation. as in previous case if CP is violeted T should also be violeted.

So have special relativity there lies lorentz invariant. resulting lorentz transformation as

$$\Lambda^{\mu}_{\nu} = \begin{pmatrix} \cosh\phi & -\sinh\phi & 0 & 0 \\ -\sinh\phi & \cosh\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Since Quantum theory allows $\phi$ to be complex so for $\phi = i\phi\pi$ then

$$\Lambda^{\mu}_{\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

which results in time reversal $t = -t$, space inversion $x \to -x$, this is a PT invariant theory. if the particle is charged then we recover the entire CPT invariance.